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Modeling of Optical Pulse Propagation in Nonlinear Dispersive Media using JE-TLM Method

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Abstract

In this paper, we propose a simulation model of electromagnetic wave's propagation in two- level atomic system. This model exploits the dependence of the polarization current density and the voltage electric in the context of the Transmission Line Matrix method with the Symmetrical Condensed Node (SCN -TLM) and novel voltage sources. By solving Maxwell's and the polarization current density equations, the proposed model, named JE-TLM, allows the simulation of the time variation of electric field and the population difference between the two energy levels of an atomic system. The scattering matrix characterizing the SCN with the new voltage sources is provided and the numerical results are compared with those of the literature or with the theoretical ones.

Keywords: Nonlinear media, JE-TLM method.

Introduction

The numerical treatment in the time domain of the propagation of an electromagnetic (EM) wave in a material medium involves frequently the use of differential methods. In the Finite difference-Time domain (FDTD) and Transmission Line Matrix (TLM) methods, it consists to resolve jointly the Maxwell's equations and the macroscopic polarization or the polarization current density equations.

The approach based on the auxiliary differential equation (ADE-FDTD) was used to characterize absorption and gain, respectively in two and four energy level atomic systems [1]. It was also used to study EM wave interaction with four-level two-electron atomic systems [2] and multi-level multi-electron atomic systems [3], taking to account the Pauli Exclusion Principle and the dynamic pumping.

The TLM method with the Symmetrical Condensed Nodes (SCN) proposed by P. B. Johns [4] has successfully simulated the behaviour of EM waves in linear and nonlinear media [5]-[6].

In this paper, we propose a novel algorithm based on the SCN-TLM method with new voltage sources. By resolving the polarization current density equation, our model allows the simulation of the effects of the media on the propagated EM wave. The time evolution of population difference between the two energy levels of an atomic system is presented.

Furthermore, the scattering matrix characterizing the SCN with the new voltage sources is provided and the simulation's results are compared to those of the literature or obtained by theoretical solutions.

Electromagnetic Wave Propagation in Dispersive Nonlinear Media

The polarization current density $\overline{J}(t)$ in a nonlinear media is linked to the electric field $E(t)$ by the equation:

$$\frac{d^2\overline{J}(t)}{dt^2}+\Delta\omega_{21}\frac{d\overline{J}(t)}{dt}+\omega_{21}{}^2\overline{J}(t) = k\Delta N_{21}(t)\frac{d\overline{E}(t)}{dt} \quad (1)$$

Where $k_a = \dfrac{6\pi\varepsilon_0 c^3}{\omega_{21}{}^2\tau_{21}}$, ε_0 is the permittivity of free space and ω_{12} is the resonant frequency of the medium. It is related to the atomic energy levels E_1 and E_2 $(E_1 < E_2)$ via the relationship $\omega_{21} = \dfrac{E_2 - E_1}{\hbar}$, $\hbar$ is the Planck constant. $\Delta\omega_{12}$ is the total energy damping factor describing the spectral width of the transition taking into account the energy loss through non radiative effects (pumping and relaxation effects). ΔN_{12} is the instantaneous difference of population describing the difference between the electron number N_1 in a state E_1 and electron number N_2 in state E_2.

Rate equations depict at each time the evolution of the population of the lower and upper levels in an atomic system taking into account the absorption, emission, induced emission, spontaneous emission and polarization current density processes.

In a atomic system with two levels $(E_1 < E_2)$, the rate equation describing the population instantaneous difference ΔN_{21} is given by:

$$\frac{d\Delta N_{21}}{dt} = -\frac{2}{\hbar\omega_{21}}\overline{E}(t)\overline{J}(t) - \frac{\Delta N - \Delta N_0}{\tau_{21}} \tag{2}$$

Where ΔN_0 is the population difference at thermal equilibrium, τ_{21} is the life duration of the electrons in the higher state E_2 and $\overline{E}(t)\overline{J}(t)$ is the instantaneous transfer energy.

JE-TLM Algorithm

In order to model nonlinear media by JE-TLM method, we should combine the polarization current density equation (1) and the rate equations (2) with Maxwell-Ampere equation given by:

$$\nabla\Lambda\overline{H} = \varepsilon_0\frac{\partial\overline{E}(t)}{\partial t} + \overline{J}(t) \tag{3}$$

Let's consider an EM wave propagating in a 3D regular mesh $(\Delta x = \Delta y = \Delta z)$, Maxwell-Ampere equations time-discretization with a time step Δt gives at a time $t^{n+1} = (n+1)\Delta t$, the following expressions:

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}^{n+1} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}^{n} - \frac{\Delta t}{\varepsilon_0}\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+\frac{1}{2}} + \frac{\Delta t}{\varepsilon_0}\begin{pmatrix} \nabla\Lambda H_x \\ \nabla\Lambda H_y \\ \nabla\Lambda H_z \end{pmatrix}^{n+\frac{1}{2}} \tag{4}$$

We replaced the EM parameters (E, H) by their equivalent voltages and currents (V, I):

$$E = V\Delta l$$
$$H = \frac{V}{(Z_0\Delta l)} \tag{5}$$

Where Δl is the space step and Z_0 is the intrinsic impedance of vacuum. Thus, the equations (5) become:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n} - \frac{\Delta l\Delta t}{\varepsilon_0}\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+\frac{1}{2}} + \frac{\Delta t\Delta l}{\varepsilon_0}\begin{pmatrix} \nabla\Lambda H_x \\ \nabla\Lambda H_y \\ \nabla\Lambda H_z \end{pmatrix}^{n+\frac{1}{2}} \tag{6}$$

The fields (E, H) in the curls of equations (6) are converted into local incident and scattered voltage pulses V^i and V^r on the faces of the SCN of the TLM Method [5]-[6]. Thus making use of charge and energy conservation principle through the transmission lines forming the SCN, and imposing the continuity conditions on the electric and magnetic fields [5], we obtain at time $n\Delta t$ the following scattering matrix of the SCN with voltage sources:

$$S = \frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0.5 \\ 0 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \end{pmatrix} \tag{7}$$

The obtained matrix models EM wave propagation in an atomic system taking into account the physical effects related with the medium polarization with the voltage sources (V_{sx}, V_{sy}, V_{sz}) which are expressed as follows:

$$\begin{pmatrix} V_{sx} \\ V_{sy} \\ V_{sz} \end{pmatrix}^{n+1} = -\begin{pmatrix} V_{sx} \\ V_{sy} \\ V_{sz} \end{pmatrix}^{n} - \frac{4\Delta l\Delta t}{\varepsilon_0}\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+\frac{1}{2}} \tag{8}$$

Where J^{n+1} is The polarization current density at time $t^{n+1} = (n+1)\Delta t$. Its expressions are deduced from the time discretization of equations (1):

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+1} = \alpha_{21}\left(\beta_{21}\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n} + \gamma_{21}\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n-1} + \frac{k}{2\Delta l}\begin{pmatrix} \Delta N_{21_x} \\ \Delta N_{21_y} \\ \Delta N_{21_z} \end{pmatrix}^{n}\left[\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} - \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n-1}\right]\right) \tag{9}$$

Where

$$\alpha_{21} = \frac{2\Delta t^2}{2+\Delta\omega_{21}\Delta t};\ \beta_{21} = \frac{2}{\Delta t^2} - \omega_{21};\ \gamma_{21} = \frac{\Delta\omega_{21}}{2\Delta t} - \frac{1}{\Delta t^2}$$

Using equation (9), and averaging across step *n* and *n*+1, give

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+\frac{1}{2}} = \frac{\alpha_{21}}{2} \left(\begin{array}{l} \left(\frac{1}{\alpha_{21}} + \beta_{21}\right) \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n} + \gamma_{21} \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n-1} \\ + \frac{k}{2\Delta l} \begin{pmatrix} \Delta N_{21_x} \\ \Delta N_{21_y} \\ \Delta N_{21_z} \end{pmatrix}^{n} \left[\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} - \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n-1} \right] \end{array} \right) \tag{10}$$

The discretized rate equation for the two-level atomic system at time $t^{n+1} = (n+1)\Delta t$ is:

$$\begin{pmatrix} \Delta N_{21_x} \\ \Delta N_{21_y} \\ \Delta N_{21_z} \end{pmatrix}^{n+1} = \delta_{21} \left(\begin{array}{l} \eta_{21} \begin{pmatrix} \Delta N_{21_x} \\ \Delta N_{21_y} \\ \Delta N_{21_z} \end{pmatrix}^{n} + \frac{\Delta N_0}{\tau_{21}} \\ - \frac{1}{2\Delta l \hbar \omega_{21}} \left[\left(\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n+1} + \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}^{n} \right) \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}^{n+\frac{1}{2}} \right] \end{array} \right) \tag{11}$$

Where

$$\delta_{21} = \frac{2\tau_{21}\Delta t}{2\tau_{21} + \Delta t}; \quad \eta_{21} = \frac{2\tau_{21} - \Delta t}{2\tau_{21}\Delta t}$$

The TLM method with voltage sources algorithm implementation is based on a recursive computation of $J^{n+1}(i, j, k)$ given by the equation (9). This allows to update voltage sources given by (8), $V^{n+1}(i, j, k)$ from (6) and to determine the population densities ΔN^{n+1} given by (11). Local scattered voltage pulses V^r at the SCN are obtained from the scattering matrix expressed in (7). Finally, we establish connections between nodes along the spatial TLM lattice.

Numerical Results

The proposed model was used to simulate the effect of an incident EM wave on a two-level atomic system. The spatial TLM lattice considered is $(1,1,5000)\Delta l$, with Δl is the mesh width taken to be $6\ nm$. The atomic system spans the cells located between $8\Delta l$ and $208\Delta l$ in the z-direction as shown by Figure 1.

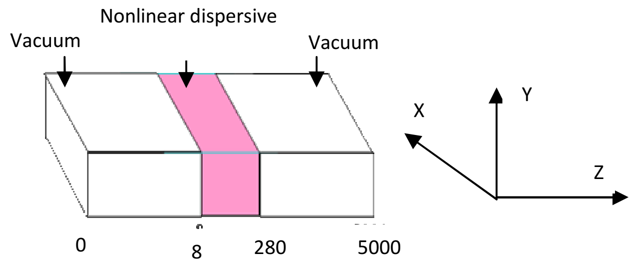


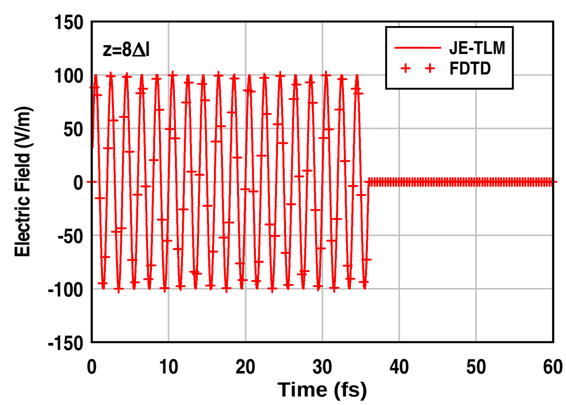
Figure 1. Treated structure.

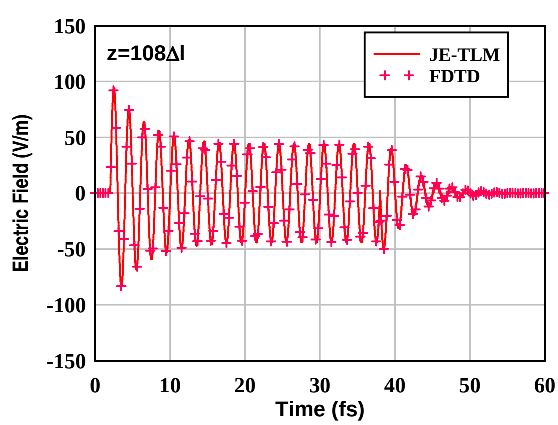
The physical parameters characterizing the two - level atomic system are:

$\omega_{12} = \pi.10^{15}\ rad/s; \Delta\omega_{12} = \pi.10^{14}\ rad/s;$
$\tau_{21} = 10^{-8}\ s$ and $\Delta N_0 = 10^{26}\ /m^3$.

We have excited the air-atomic system interface using a square wave of amplitude $100\ V/m$, frequency $5.10^{14}\ Hz$ and overall duration 36fs [1].

Figure 2 shows the evolution of low intensity pulse (linear case), during its propagation in the medium, at the centre, the air-system and system-air interfaces.





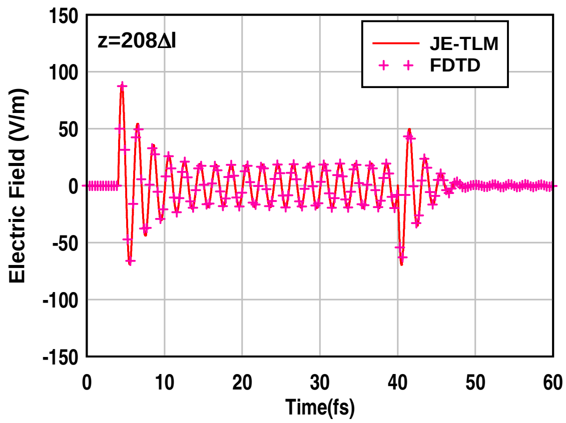


Figure 2. Transient response of the linear media to a square pulse simulated by JE-TLM and FDTD methods.

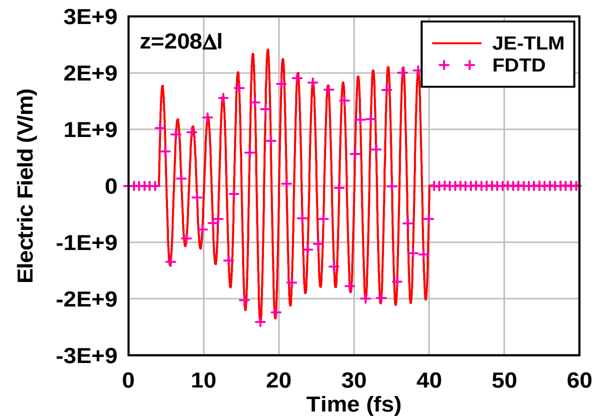
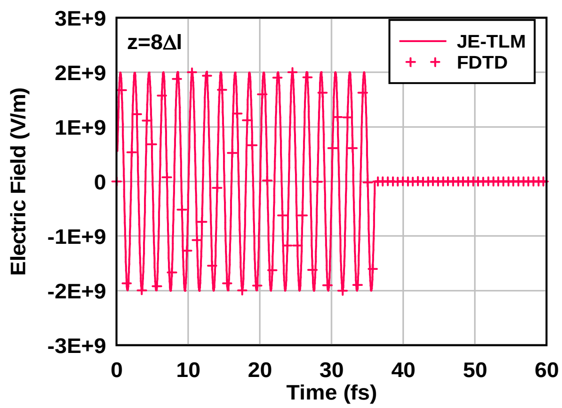
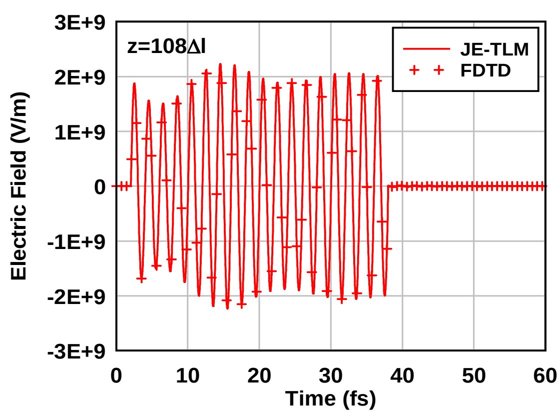


Figure 3. Transient response of the nonlinear media to a square pulse calculated by JE-TLM and FDTD methods.

Next, we excite the air-system by a strong pulse of amplitudem $2.10^9 V/m$, frequency 5.10^{14} Hz and overall duration 36fs [1].

Figure 3 illustrates the nonlinear response of the medium in the presence of an electric field amplitude incident strong, observed at same lecture points as Figure 2.





The medium is absorbent; the reflected field tend to reduce the magnitude of the total field in both linear and nonlinear cases. Therefore, the pulse undergoes a reduction in its propagation along the media.

Figure 4 shows the impact of the incident field with amplitude $2.10^9 V/m$ and frequency 5.10^{14} Hz on the two-level atomic system. As a consequence, it was established that population difference tends to zero: this is the situation of the EM transition. . It even becomes negative implying that the population inversion has taken place.

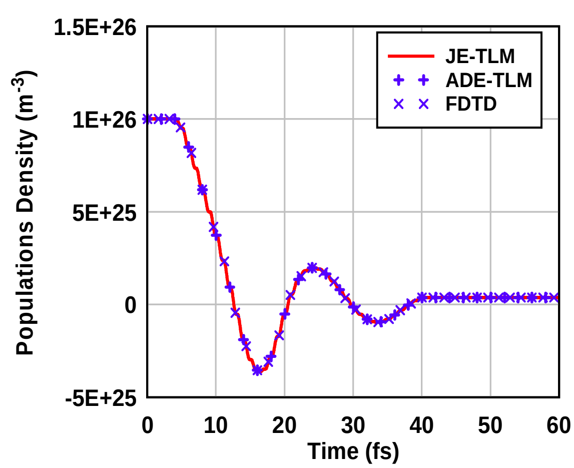


Figure 4. Comparison of population difference ΔN_{21} at the interface material-air as computed by JE-TLM, ADE-TLM [7] and FDTD methods.

Finally, the air-nonlinear dispersive medium interface is exited by Gaussian unit amplitude pulse. Figure 5 compares the absorption factor as computed by theory, JE-TLM, ADE-TLM and FDTD methods.

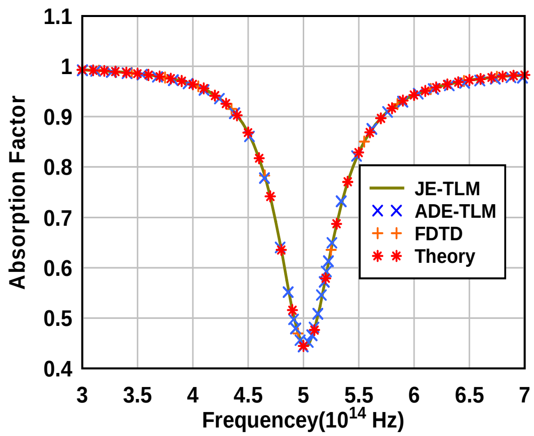


Figure 5. Comparison of absorption factor versus frequency as computed by theory, JE-TLM, ADE-TLM [7] and FDTD methods.

Conclusion

In this paper, we have modelled the interaction EM waves, of low and strong intensities, with two-level atomic systems. The JE-TLM method has allowed studying the electrons transition between two energy levels, by solving the rate, the polarization current density and Maxwell's equations.

The good agreement between the novel JE-TLM approach results and those available in the literature proves its validity and efficiency.

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